Space-time versus particle-hole symmetry in quantum Enskog equations

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The nonlocal scattering-in and scattering-out integrals of the Enskog equation have reversed displacements of colliding particles reflecting that the scattering-in and -out processes are conjugated by the space and time inversions. Generalizations of the Enskog equation to Fermi liquid systems are hindered by the need for particle-hole symmetry which contradicts the reversed displacements. We resolve this problem with the help of the optical theorem. It is found that space-time and particle-hole symmetry can be fulfilled simultaneously only for the Bruckner type of internal Pauli blocking while the Feynman-Galitskii form allows only for particle-hole symmetry but not for space-time symmetry due to a stimulated emission of bosons.

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I. INTRODUCTION

The formulation of kinetic theory of dense interacting Fermi gases beyond the Boltzmann equation (BE) is an ongoing task. For a classical hard-sphere gas the main theoretical focus has been on the statistical correlations resulting in the Enskog equation [1–7]. In contrast to the BE, the collision integral of the Enskog equation is nonlocal; it takes into account that, when two hard spheres collide, their centers are displaced by the sum of their radii. The particle scattered out of its free trajectory faces its collision partner in front, while the particle scattered in the new free trajectory leaves its partner behind. This is expressed by the opposite signs of nonlocal corrections in the scattering-out and scattering-in integrals.

Various generalizations of the Enskog equation toward quantum systems [8–14] have been developed mostly in the last two decades. They offer numerous gradient corrections to the scattering integral which describe how the nonlocal character of collisions contributes to smooth perturbations. With a typical number of gradient corrections counted in tens, a comparison of the original Enskog equation with its generalizations was not possible.

The connection became more clear after Tastevin, Nacher, and Laloe [13] recognized that some of the gradient corrections obtained can be recast into effective fields and renormalizations of the mass of particles, i.e., these gradient corrections are linked to the Landau concept of quasiparticles. They also show that when the quasiparticle contributions are separated, all remaining gradient contributions are proportional to various derivatives of the scattering phase shift. These derivatives have a natural link to the Wigner collision delay [15] which also describes the nonlocality of collisions, although in time not in space.

A kinetic equation that combines the nonlocality in time and space has been derived as the quasiclassical asymptotics of nonequilibrium Green's functions [16,17]. In [16] a backward resummation of the gradient expansion was introduced by which one obtains the scattering integral in a form recalling the Enskog equation: the gradient corrections are expressed as shifts of arguments in the initial (final) condition so that one can see how long the collision lasts and how far from each other the particles are at the beginning (end) of a collision. Of course, the hard-sphere gas is a special case to which the theory applies. It turns out that the scattering in is identical to the Enskog equation while the displacement of the scattering out does not have the expected opposite sign. A careful inspection shows that this sign problem appears also within all earlier approaches [8–14].

The sign puzzle has two serious consequences for the applicability of the nonlocal kinetic equation. First, the Enskog equation corresponds to classical trajectories; therefore it can be numerically studied either with a Monte Carlo simulation or by the so called molecular dynamics. The kinetic equations derived from quantum statistics cannot be studied with these methods. Second, the Enskog equation yields the hydrodynamic Chapman-Enskog expansion in a straightforward and relatively simple manner [2] which allows one to identify the thermodynamic properties of the system. The symmetry between the scattering in and out is a very important prerequisite in separation of canceling and conserving quantities. Without this symmetry, one can also derive conservation laws [12]; however, an extensive application of physically nontransparent identities is necessary.

In this paper we show how the natural symmetry of the Enskog equation can be obtained within the quantum mechanical approach to the kinetic equation. In the next section we introduce the problem of symmetry in a naive manner using *ad hoc* kinetic equations for the Fermi liquid. In Sec. III we show that the nonlocal corrections for the Fermi liquid are linked to in-medium effects and provide an identity that allows one to achieve the Enskog form of nonlocal corrections. Sec. IV includes conclusions.

II. CLASSICAL VERSUS QUANTUM COLLISION

The problem with sign in the scattering out follows from a difference between the classical and quantum approaches to collisions. One has to recognize that a realistic collision has a finite duration Δ_t and to compare these two approaches in the time picture.

A. Pseudoclassical approach

The simplest model system on which one can illustrate both approaches is a homogeneous gas of particles that form

short-living molecules, i.e., the system with dominant resonant scattering [15]. Its kinetic equation reads

$$\frac{\partial f_k(t)}{\partial t} = \int P F_{k+p}(t) - \int P f_k(t) f_p(t). \tag{1}$$

The last term is the scattering out which describes that with the probability P two particles form a molecule and thus a particle leaves the state of momentum k. The first term on the right hand side corresponds to the decay of the molecule into two particles, one of them achieves momentum k. The dependence of the distribution of molecules F is also covered by the balance equation

$$\frac{\partial F_K(t)}{\partial t} = \int P f_{K-p}(t) f_p(t) - \frac{F_K(t)}{\Delta_t}. \tag{2}$$

The last term describes the decay of molecules with lifetime Δ_t, the first term on the right hand side their formation.

The balance equation (2) for molecules is solved by

$$F_K(t) = \int_0^\infty d\tau\, e^{-\tau/\Delta_t} \int P f_{K-p}(t-\tau) f_p(t-\tau)$$

$$\approx \int P f_{K-p}(t-\Delta_t) f_p(t-\Delta_t). \tag{3}$$

The second line is the gradient approximation which is sufficient for our discussion since all quantum approaches to the nonlocal kinetic equation are restricted to it. Using Eq. (3) in Eq. (1) one gets a kinetic equation,

$$\frac{\partial f_k(t)}{\partial t} = \int P f_{k-q}(t-\Delta t) f_{p+q}(t-\Delta_t) - \int P f_k(t) f_p(t). \tag{4}$$

The scattering in has a retarded initial condition reflecting that the molecule exists from $t-\Delta_t$ to t. The initial condition of the scattering out is associated with time instant t; the corresponding molecule thus exists from t to $t+\Delta_t$.

In dense Fermi systems, the final states of collisions might be occupied and the collision is then prohibited. Let us modify kinetic equation (4) by *ad hoc* Pauli blocking factors as introduced by Nordheim [18] and by Uehling and Uhlenbeck [19]

$$\frac{\partial f_k(t)}{\partial t} = \int P f_{k-q}(t-\Delta_t) f_{p+q}(t-\Delta_t)[1-f_k(t)][1-f_p(t)]$$

$$- \int P f_k(t) f_p(t)[1-f_{k-q}(t+\Delta_t)]$$

$$\times [1-f_{p+q}(t+\Delta_t)]. \tag{5}$$

The time arguments of the blocking factors $1-f$ correspond to the ends of the time intervals during which the collision happens because the blocking is attributed to the final states.

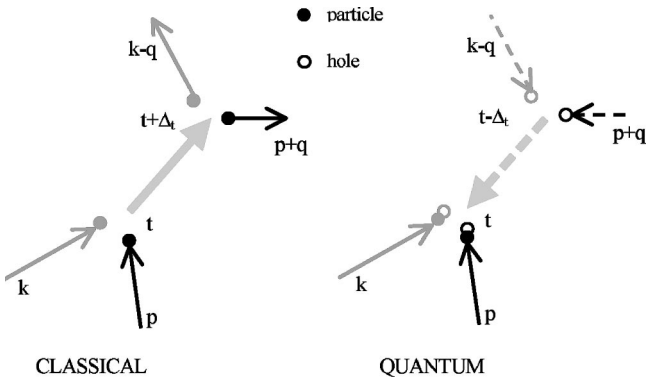

FIG. 1. Scattering out for the classical and quantum concepts of collisions.

B. Quantum approach

One can see that the scattering out of Eq. (5), obtained within pseudoclassical assumptions, requires the blocking factor at future time $t+\Delta_t$. The quantum approach, however, does not allow one to look into the future and treats the same process differently.

In the quantum statistics, the scattering out is described as a collision of two holes; see Fig. 1. In our simple model, two holes form a hole-molecule which also exists for Δ_t. When this hole-molecule decays into two holes, these holes annihilate particles of corresponding momenta. Accordingly, the scattering out is described by the hole-hole interaction during the time interval from $t-\Delta_t$ to t. An *ad hoc* kinetic equation corresponding to the quantum picture thus reads

$$\frac{\partial f_k(t)}{\partial t} = \int P f_{k-q}(t-\Delta_t) f_{p+q}(t-\Delta_t)[1-f_k(t)][1-f_p(t)]$$

$$- \int P f_k(t) f_p(t)[1-f_{k-q}(t-\Delta_t)]$$

$$\times [1-f_{p+q}(t-\Delta_t)]. \tag{6}$$

Note that Eq. (6) differs from its pseudoclassical counterpart (5) by the sign of the nonlocal correction. This is the time modification of the sign problem found for the quantum generalizations of the Enskog equation.

The above *ad hoc* implementations, Eqs. (5) and (6), of the nonlocal corrections reveal a paradox: The space-time symmetry and the particle-hole symmetry lead to contradictory results. Indeed, Eqs. (5) and (6) are different and for a general scattering rate P they correspond to different thermodynamic properties of the system.

III. IN-MEDIUM EFFECTS

To resolve the paradox of symmetries, one has to take into account that the scattering rate P itself is a function of the occupation, $P[f]$. This dependence represents an internal Pauli blocking of states during collisions, which is called the in-medium effect in nuclear physics. In a heuristic manner one can indicate what kind of internal Pauli blocking is consistent with the Uehling-Uhlenbeck blocking of final states.

Since the scattering process lasts over the time interval

from $t-\Delta_t$ to t, the mean value of P equals its value at the center time $P=P(t-1/2\Delta_t)$; see [16,17]. Comparing Eqs. (5) and (6) we find that the two symmetries are consistent if

$$\int P\left(t-\frac{1}{2}\Delta_t\right) f_k(t) f_p(t)[1-f_{k-q}(t-\Delta_t)] \times[1-f_{p+q}(t-\Delta_t)] = \int P\left(t+\frac{1}{2}\Delta_t\right) f_k(t) f_p(t)[1-f_{k-q}(t+\Delta_t)] \times[1-f_{p+q}(t+\Delta_t)]. \quad (7)$$

Within the gradient approximation this condition reads

$$\int P f_k f_p (1-f_{k-q})(1-f_{k-q}) \times\left(\frac{d\ln P}{dt}+2\frac{d}{dt}\ln[(1-f_{k-q})(1-f_{p+q})]\right)=0. \quad (8)$$

From this equation we see that the space-time and particle-hole symmetries are consistent when the time dependence of the in-medium effect is given by Pauli blocking of Bruckner type, $P[(1-f)(1-f)]$. According to this type of Pauli blocking, the internal states of the short-living molecule exist only in the unoccupied phase space.

A. Causality

The retarded scattering-out integral of Enskog type (5) is peculiar from the point of view of the causality. Since the scattering-out process ends at time $t+\Delta_t$, the scattered particles have to have an available final state at this time. In other words, to determine whether the collision is allowed by the Pauli exclusion principle, one has to look into the future. In this way, the Pauli blocking seems to create an anticasual step.

In general, the causality of the perturbative expansion reflects the tendency of a many-body system to reach its equilibrium state. An anticausal description of the whole system is thus impossible because of the dissipative processes. Accordingly, we will take the causal expansion and the subsequent particle-hole symmetry represented by Eq. (6) as a well justified starting point.

The Enskog-type kinetic equation with the space-time symmetry of the scattering integral applies only under restrictive assumptions. The first assumption is that individual binary collisions are treated as if they were isolated from the rest of the system. The dynamics of the binary collision is then reversible and the causal and anticausal expansions on the space-time scale of a single collision are equivalent. This assumption is met in all approaches to the kinetic equation except for the studies of the so called collisional broadening. The second assumption is that the internal Pauli blocking of collisions is of the Bruckner type. This point is discussed below.

All methods of quantum statistics enforce the causality using backward propagation of holes instead of the forward propagation of particles into the future. To link the space-time and particle-hole symmetries, we will use the optical theorem which allows us to reformulate the causal internal propagation during the collision into the anticausal one.

In the algebraic notation of the double-time Green functions [20–22], the causality is reflected by the order of operators retarded-correlation-advanced. The time cuts of the retarded and advanced operators restrict all time integrals to the past. The anticausal expansion is then characterized by the reversed order advanced-correlation-retarded. Without introducing unnecessary details, we can link the causal and anticausal expansions using the identity for the scattering T matrix,

$$T^R\mathcal{A}T^A=T^A\mathcal{A}T^R. \quad (9)$$

This identity represents two forms of the optical theorem, $\mathrm{Im}\,T=T^R\mathcal{A}T^A$ and $\mathrm{Im}\,T=T^A\mathcal{A}T^R$. Their derivations are in Appendix A.

The retarded/advanced T matrix $T^{R,A}$ describes an individual binary process [23]. The two-particle spectral function $\mathcal{A}$ includes the internal Pauli blocking. In this paper we discuss two particular approximations of the internal Pauli blocking, the Bruckner approximation,

$$\mathcal{A}_B(t_1,t_2,k,p)\approx(1-f_k)(1-f_p)e^{-i(\epsilon_k+\epsilon_p)(t_1-t_2)}, \quad (10)$$

and the Galitskii-Feynman approximation,

$$\mathcal{A}_{GF}(t_1,t_2,k,p)\approx(1-f_k-f_p)e^{-i(\epsilon_k+\epsilon_p)(t_1-t_2)}. \quad (11)$$

For both approximations we will derive kinetic equations with the space-time symmetry of the nonlocal scattering integral. We will see that the kinetic equation obtained within the Bruckner approximation has the pseudoclassical form (5); therefore it can be treated with numerical tools based on the classical concept of trajectories. In contrast, the kinetic equation within the Galitskii-Feynman approximation includes a nontrivial term due to the stimulated emission of bosons which essentially complicates its numerical treatment.

B. Collision integral from Green functions

Let us first recall how the nonlocal scattering integrals relate to more general relations of quantum statistics. We demonstrate it on the method of nonequilibrium Green functions. The scattering-in and -out integrals result from anticommutators $\{.,.\}$ of the Kadanoff and Baym (KB) equation [24–26]

$$\{G^>,\Sigma^<\}-\{G^<,\Sigma^>\}=\{G^>,G^>\circ T^R(G^<G^<)T^A\} -\{G^<,G^<\circ T^R(G^>G^>)T^A\}. \quad (12)$$

Here, $G^<$ and $G^>$ are particle and hole correlation functions; the $\circ$ denotes that $G^{>,<}$ closes one loop of the two-particle function on its right hand side. The T matrices and pairs of

single-particle correlation functions (GG) obey standard two-particle operator products.

With respect to our treatment, it is sufficient to know that the particle correlation functions are proportional to the quasiparticle distribution $G^<(t_1,t_2,k)\approx f_k e^{-i\epsilon_k(t_1-t_2)}$; therefore they represent the initial states of collisions. Similarly, the hole correlation functions are proportional to the hole distribution $G^>(t_1,t_2,k)\approx(1-f_k)e^{-i\epsilon_k(t_1-t_2)}$; therefore they describe the final states including their Pauli blocking.

According to the initial and final states, the first and second terms of the right hand side of Eq. (12) can be interpreted as the scattering-in and scattering-out integrals, respectively. Note that both scattering integrals are causal, having the order retarded-correlation-advanced of the two-particle functions. At the same time, the scattering-in and scattering-out integrals are linked via the particle-hole symmetry. One can see that upon the interchange of particles and holes, $>\leftrightarrow<$, the first term changes to the second one and vice versa. Equation (12) is thus a precursor of Eq. (6).

Using the (extended) quasiparticle and quasiclassical approximations and keeping gradients in the scattering integral of the KB equation, one obtains a nonlocal kinetic equation [16],

$$\frac{\partial f_1}{\partial t}+\frac{\partial \epsilon_1}{\partial k}\frac{\partial f_1}{\partial r}-\frac{\partial \epsilon_1}{\partial r}\frac{\partial f_1}{\partial k}=\int P^- f_3^- f_4^-(1-f_1)(1-f_2^-) - \int P^-(1-f_3^-)(1-f_4^-)f_1 f_2^-. \tag{13}$$

Algebraic operations needed to arrive at Eq. (13) are rather extensive due to numerous gradient contributions to the scattering integrals. These gradient contributions are expressed via shifts of arguments as

$$f_1\equiv f(k,r,t),$$

$$f_2^-\equiv f(p,r-\Delta_2,t),$$

$$f_3^-\equiv f(k-q-\Delta_K,r-\Delta_3,t-\Delta_t),$$

$$f_4^-\equiv f(p+q-\Delta_K,r-\Delta_4,t-\Delta_t). \tag{14}$$

The differential cross section is proportional to the square of the amplitude of the T matrix

$$P^-=\frac{dp}{(2\pi)^3}\frac{dq}{(2\pi)^3}2\pi\delta(\epsilon_1+\epsilon_2^--\epsilon_3^--\epsilon_4^--2\Delta_E) \times\left|T\left(\epsilon_1+\epsilon_2^--\Delta_E,k-\frac{\Delta_K}{2},p-\frac{\Delta_K}{2},q,r-\Delta_r,t-\frac{\Delta_t}{2}\right)\right|^2. \tag{15}$$

The arguments of the quasiparticle energies ϵ are identical with Eq. (14). All nonlocal corrections are given by derivatives of the scattering phase shift $\phi=\mathrm{Im}\ln T^R(\Omega,k,p,q,t,r)$ [16],

$$\Delta_t=\frac{\partial\phi}{\partial\Omega},\quad \Delta_E=-\frac{1}{2}\frac{\partial\phi}{\partial t},\quad \Delta_K=\frac{1}{2}\frac{\partial\phi}{\partial r},\quad \Delta_3=-\frac{\partial\phi}{\partial k},$$

$$\Delta_2=\frac{\partial\phi}{\partial p}-\frac{\partial\phi}{\partial q}-\frac{\partial\phi}{\partial k},\quad \Delta_4=-\frac{\partial\phi}{\partial k}-\frac{\partial\phi}{\partial q}. \tag{16}$$

A detailed understanding of these numerous corrections to the scattering integral of the Boltzmann equation is not essential for our discussion. It is important to realize that the collision is of finite duration Δ_t. During this time particles can gain momentum and energy $\Delta_{K,E}$ due to the medium effect on the collision. Three displacements $\Delta_{2,3,4}$ correspond to initial and final positions of two colliding particles/holes.

The quasiparticle kinetic equation (13) covers three ingredients of the kinetic theory. First, the scattering integral includes the medium effect on the scattering rate. Second, the scattering integrals are nonlocal in space and time. Third, the quasiparticle energy represents the momentum dependent mean field. With respect to the included nonlocal corrections, it is important that the quasiparticle energy is defined from the pole of the propagator, not from the variation of the energy density. This difference has been discussed in [27]. The scattering out of Eq. (13) is the particle-hole mirror of the scattering in; accordingly it is not the space-time mirror found in the Enskog equation.

C. Anticausal collision integral

Our aim is to rearrange (13) so that it will include the scattering out as the space-time mirror of the scattering in; briefly, it will be the symmetry assumed by Enskog. It is advantageous to make this step already on the level of Green functions. Accordingly, we rearrange Eq. (12) so that its scattering-out part is written in terms of the anticausal expansion.

Further progress depends on the approximation of the T matrix. Let us first approximate the T matrix by Bruckner's reaction matrix for which the two-particle spectral function is $\mathcal{A}_B=(G^>G^>)$. Formula (10) is the quasiparticle approximation of $(G^>G^>)$. Using identity (9) in the second term of Eq. (12) one finds

$$\{G^>,\Sigma^<\}-\{G^<,\Sigma^>\}=\{G^>,G^>\circ T^R(G^<G^<)T^A\} -\{G^<,G^<\circ T^A(G^>G^>)T^R\}. \tag{17}$$

Expression (17) has the desired explicit space-time symmetry in contrast to the explicit particle-hole symmetry of Eq. (12). To see it in detail, we use Eq. (17) in the KB equation and employ the same steps as above (quasiclassical and quasiparticle approximations) to arrive at

$$\frac{\partial f_1}{\partial t}+\frac{\partial \epsilon_1}{\partial k}\frac{\partial f_1}{\partial r}-\frac{\partial \epsilon_1}{\partial r}\frac{\partial f_1}{\partial k}=\int P^- f_3^- f_4^-(1-f_1)(1-f_2^-) - \int P^+(1-f_3^+)(1-f_4^+)f_1 f_2^+. \tag{18}$$

The shifts in the scattering out have opposite signs,

$$f_2^+ \equiv f(p, r+\Delta_2, t),$$

$$f_3^+ \equiv f(k-q+\Delta_K, r+\Delta_3, t+\Delta_t),$$

$$f_4^+ \equiv f(p+q+\Delta_K, r+\Delta_4, t+\Delta_t), \tag{19}$$

and similarly other ingredients denoted by superscript $+$.

It can be indicated why the change of the causal picture into the anticausal one results in the flipped signs of all nonlocal corrections. First, one can use a formal argument. Writing the T matrices as products of the amplitude and the phase, $T^R = |T| e^{i\phi}$ and $T^A = |T| e^{-i\phi}$, one can see that the interchange of retarded and advanced T matrices merely flips the sign of the phase shift ϕ. As all Δ's depend linearly on ϕ, the gradient contributions of the anticausal scattering out have signs reversed with respect to the causal scattering in. Second, there is a physical reason for the formal argument above. The amplitude of the T matrix represents a filter which selects the probability of individual channels. The factor of the phase shift $e^{i\phi}$ is a unitary transformation which applies to individual components of the wave function in a manner that parallels the evolution operator. Products like $e^{i\phi} \cdots e^{-i\phi}$ correspond to transformation from one place to another, and $e^{-i\phi} \cdots e^{i\phi}$ to the backward one.

For the system of classical hard spheres, the kinetic equation (18) reduces to the Enskog equation in the second order virial approximation. This limit includes three simplifications. First, the Pauli blocking factors vanish in the classical limit, $1-f \to 1$. Second, the quasiparticle energy reduces to the kinetic energy of free particles, $\epsilon_1 \to k^2/2m$. For this limit it is important that the quasiparticle energy is defined from the pole of the propagator. Landau's definition of ϵ based on the variation of the energy density yields a nontrivial quasiparticle energy even for the classical gas of hard spheres. Third, from the hard-sphere scattering phase shift, $\phi \to \pi - |q| D$, where D is the diameter of colliding particles, one finds the expected values of the Δ's. The collision delay is zero, $\Delta_t \to 0$, there is no energy/momentum gain, $\Delta_{E,K} \to 0$, during collision, and none of the particles move in space, $\Delta_3 = 0$ and $\Delta_4 = \Delta_2$. The displacement of particles at the instant of collision is $\Delta_{2,4} = D$.

To summarize this section, we have shown that the space-time and the particle-hole symmetric forms of the nonlocal Boltzmann equation are equivalent if the scattering rate includes the in-medium effect on the level of the Bruckner reaction matrix.

D. Comments on the Galitskii-Feynman *T* matrix

The Bruckner approximation of the scattering rate was quite common in earlier microscopic studies of heavy ion reactions. Recently, most studies prefer the Galitskii-Feynman approximation [25,26] for which the Pauli blocking of the internal states is controlled by the two-particle spectral function, $\mathcal{A}_{GF} = (G^> G^>) - (G^< G^<)$. Formula (11) is the quasiparticle approximation of $\mathcal{A}_{GF}$.

As mentioned, the Galitskii-Feynman approximation includes processes that go beyond the scope of naive kinetic equations. While terms proportional to $(G^> G^>)$ exclude correlation in the occupied phase space, terms proportional to $(G^< G^<)$ describe stimulated correlation by already existing pairs. These processes lead to the superconducting phase transition at low temperatures; therefore the system cannot be treated as a sum of single-particle excitations on the Fermi liquid ground state. Kinetic equations (5) and (6) are based on the idea of the simple Fermi liquid and do not include the stimulated processes.

Even with the stimulated processes included, the kinetic equation can be rearranged into the space-time symmetric form. Again, we make the rearrangement on the level of the Green functions. The scattering integrals (12) can be written so that their final states are consistent with the Galitskii-Feynman spectral function,

$$\begin{aligned} \{G^>, \Sigma^<\} - \{G^<, \Sigma^>\} = & \{G^>, G^> \circ T^R (G^< G^<) T^A\} \\ & - \{G^<, G^< \circ T^R (G^< G^<) T^A\} \\ & - \{G^<, G^< \circ T^R ((G^> G^>) \\ & - (G^< G^<)) T^A\}. \end{aligned} \tag{20}$$

The last term includes the Galitskii-Feynman two-particle spectral function and can be converted into the anticausal picture. The causal/anticausal forms of the resulting kinetic equation read,

$$\begin{aligned} \frac{\partial f_1}{\partial t} + \frac{\partial \epsilon_1}{\partial k} \frac{\partial f_1}{\partial r} - \frac{\partial \epsilon_1}{\partial r} \frac{\partial f_1}{\partial k} = & \int P^- \; f_3^- f_4^- (1 - f_1 - f_2^-) \\ & - \int P^\mp (1 - f_3^\mp - f_4^\mp) f_1 f_2^\mp . \end{aligned} \tag{21}$$

The particle-hole symmetric form (with superscripts $-$) can be recast into Eq. (6) by a subtraction of stimulated processes, proportional to $f_3^- f_4^- f_1 f_2^-$ on both sides. In contrast, the space-time symmetric form (with superscripts $+$) cannot be recast into the intuitive form (5) due to gradient contributions of stimulated processes, proportional to $f_3^+ f_4^+ f_1 f_2^+$. It is a pity, since the space-time symmetry is obligatory for numerical treatments based on Monte Carlo simulations. Equation (21) is not suited for the Monte Carlo treatment because its scattering integrals can change their sign, losing their probabilistic interpretation. The Galitskii-Feynman type of kinetic equation (21) thus provides a more precise description of the system, but at the cost of a serious increase in difficulties of its numerical treatment. Because of these numerical problems, we discuss implementation of symmetries only for the Bruckner approximation. A comparison between the Bruckner and Galitskii-Feynman approximations can be found in [28] showing that the latter can describe the onset of pairing in contrast to the Bruckner approximation. About the range of validity of the Bruckner approximation, see [29] and citations therein.

E. Implementation of the symmetry in simulations

The equivalency of both forms of the kinetic equation, (13) and (18), offers an important simplification of the numerical treatment. Expanding the scattering out to the linear terms, one finds that amplitudes of anticausal and causal corrections are equal while the signs are opposite. Since both forms are equivalent, the sum of gradient corrections to the scattering out vanishes.

For highly inhomogeneous and/or fast evolving systems, like nuclear matter in a heavy ion reaction, the Monte Carlo simulation procedure spends a majority of the CPU time searching when and where a collision should be generated. Due to cancellation of gradient corrections to the scattering-out integral, this part of the simulation procedure remains the same as in the local approximation. All nonlocal corrections are included only after the collision event is selected. This scheme was used in [30].

IV. CONCLUSIONS

We have shown that the space-time symmetry of the nonlocal scattering integral becomes nontrivial if the Pauli exclusion principle has to be accounted for. Within the pseudoclassical form of the Pauli blocking represented by the hole distributions as introduced by Nordheim and Uehling and Uhlenbeck, the space-time symmetry and the particle-hole symmetry are consistent only if the scattering cross section includes in-medium effects of Bruckner type. Due to their classical form, these nonlocal corrections are easily implemented into the Monte Carlo simulations.

The more sophisticated approximation of Galitskii and Feynman includes the stimulated creation of the colliding pair. This process escapes the pseudoclassical interpretation of the scattering process which makes its implementation within the traditional Monte Carlo simulation schemes impossible.

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APPENDIX: OPTICAL THEOREM

Identity (9) represents two alternative expressions of the anti-Hermitian part of the T matrix,

$$M = \mathrm{Im}\, T = i(T^R - T^A). \tag{A1}$$

We derive this identity known as the optical theorem from the ladder approximation, which is the approximation used in the scattering integrals of the discussed kinetic equation.

The ladder approximation in the differential form reads

$$T^{-1}_{R,A} = V - \mathcal{G}^{R,A}. \tag{A2}$$

Here, $\mathcal{G}^{R,A}$ are the two-particle propagators given by the time cut of the spectral function $\mathcal{A} = i(\mathcal{G}^R - \mathcal{G}^A)$. From Eq. (A2) follows

$$i(T_R^{-1} - T_A^{-1}) = -\mathcal{A}. \tag{A3}$$

Multiplying Eq. (A1) by T_R^{-1} one finds

$$T_R^{-1} M = i - i T_R^{-1} T_A. \tag{A4}$$

Finally, we express T_R^{-1} from Eq. (A3),

$$T_R^{-1} = T_A^{-1} + i\mathcal{A}, \tag{A5}$$

so that Eq. (A4) turns into the familiar optical theorem

$$M = T^R \mathcal{A} T^A. \tag{A6}$$

To obtain a less familiar anticausal form of the optical theorem, we multiply Eq. (A1) by T_R^{-1} from the right hand side,

$$M T_R^{-1} = i - i T_A T_R^{-1}. \tag{A7}$$

Now we substitute Eq. (A5) into Eq. (A7) which yields

$$M = T^A \mathcal{A} T^R. \tag{A8}$$

Comparing Eq. (A6) with Eq. (A8) one obtains identity (9).

[1] D. Enskog, in *Kinetic Theory*, edited by S. Brush (Pergamon Press, New York, 1972), Vol. 3 [originally K. Sven. Vetenskapsakad. **63**, No. (4) (1921)].
[2] S. Chapman and T. G. Cowling, *The Mathematical Theory of Non-uniform Gases*, 3rd ed. (Cambridge University Press, Cambridge, 1990), Chap. 16.
[3] E. Cohen, *Fundamental Problems in Statistical Mechanics* (North-Holland, Amsterdam, 1962).
[4] J. Weinstock, Phys. Rev. **140**, A460 (1965).
[5] K. Kawasaki and I. Oppenheim, Phys. Rev. **139**, A1763 (1965).
[6] J. R. Dorfman and E. G. Cohen, J. Math. Phys. **8**, 282 (1967).
[7] H. van Beijeren and M. H. Ernst, J. Stat. Phys. **21**, 125 (1979).
[8] K. Bärwinkel, Z. Naturforsch. A **24**, 38 (1969).
[9] F. Laloë, J. Phys. (Paris) **50**, 1851 (1989).
[10] D. Loos, J. Stat. Phys. **59**, 691 (1990).
[11] R. Snider, J. Stat. Phys. **61**, 443 (1990).
[12] M. de Haan, Physica A **164**, 373 (1990).
[13] P. J. Nacher, G. Tastevin, and F. Laloë, Ann. Phys. (Leipzig) **48**, 149 (1991).
[14] T. Bornath, D. Kremp, W. D. Kraeft, and M. Schlanges, Phys. Rev. E **54**, 3274 (1996).
[15] P. Danielewicz and S. Pratt, Phys. Rev. C **53**, 249 (1996).
[16] V. Špička, P. Lipavský, and K. Morawetz, Phys. Lett. A **240**, 160 (1998).
[17] P. Lipavský, K. Morawetz, and V. Špička, Ann. Phys. (Paris) **26**, 1 (2001); K. Morawetz, Habilitation, University Rostock, 1998.

[18] L. W. Nordheim, Proc. R. Soc. London, Ser. A **119**, 689 (1928).
[19] E. A. Uehling and G. E. Uhlenbeck, Phys. Rev. **43**, 552 (1933).
[20] L. V. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964).
[21] D. C. Langreth and J. W. Wilkins, Phys. Rev. B **6**, 3189 (1972).
[22] L. Banyai and K. E. Sayed, Ann. Phys. (N.Y.) **233**, 165 (1994).
[23] M. L. Goldberger and K. M. Watson, *Collision Theory* (Wiley, New York, 1964).
[24] L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, New York, 1962).
[25] P. Danielewicz, Ann. Phys. (N.Y.) **152**, 239 (1984).
[26] W. Botermans and R. Malfliet, Phys. Rep. **198**, 115 (1990).
[27] P. Lipavský, V. Špička, and K. Morawetz, Phys. Rev. E **59**, R1291 (1999).
[28] T. Alm *et al.*, Phys. Rev. C **53**, 2181 (1996).
[29] H. S. Köhler and R. Malfliet, Phys. Rev. C **48**, 1034 (1993).
[30] K. Morawetz *et al.*, Phys. Rev. Lett. **82**, 3767 (1999).